

# Numbers

## ...with Ramanujan

### UNIT 1

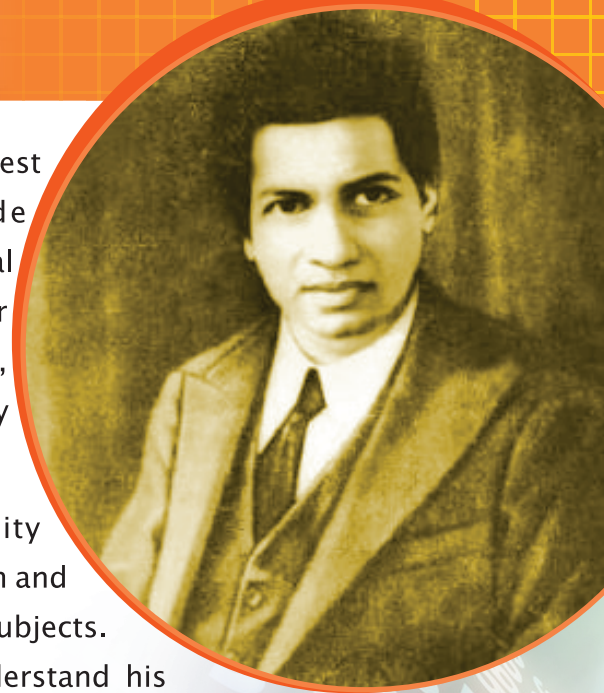
### Chapter

# 1

Ramanujan was one of India's greatest mathematical geniuses who made substantial contribution to the analytical theory of numbers. He was born in the year 1887 and died in 1920. As a person, Ramanujan was shy and quiet but very dignified and well mannered.

Ramanujan had no formal university education; however, he studied on his own and carried out research work on various subjects.

Unfortunately, most people did not understand his theories and hence, he made very little money. Despite this, he continued his research work till the very end of his short life.



**Hi! I am Ramanujan.**

**We use numbers so often in our daily lives; we use them while telling the time or the cost of something. So, don't you think we should be good at numbers to convey things easily?**

**Besides, if you're good at numbers and if you want, you could become a data analyst!**

**A data analyst goes through volumes of numbers to find relevant data, summarises this data and arranges it in such a manner that it can be used to draw conclusions.**

**So, come with me. Let's start our adventure to the world of numbers!**



## What you already know...



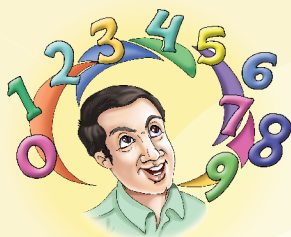
The place at which a digit occurs in a number determines its place value. Let me help you recall the Indian place value system. If I had to write the number 2,34,56,789, here's how I would place it in the Indian place value system.

Crores	Ten lakhs	Lakhs	Ten thousand	Thousand	Hundreds	Tens	Ones
2	3	4	5	6	7	8	9

The place value of 2 is 2,00,00,000

The place value of 3 is 30,00,000

The place value of 4 is 4,00,000 and so on



**I have some data that a data analyst had asked me to arrange. I will be impressed if you can do this little activity for me!**

Some numbers have been given in your follow-up journal. Write the numbers in the increasing order of their values. Then, write them in the increasing order of the place values of 8. Do this in the table given in your follow-up journal.

## What you need to know...



**You know you live in a fascinating age, where you have so many vocations to choose from. In my time, we did not have so many choices! If I was born in the present time and age, I would have loved to be a data analyst, sitting comfortably at a computer and crunching numbers! Numbers are indeed fascinating. Let us get to know numbers better.**

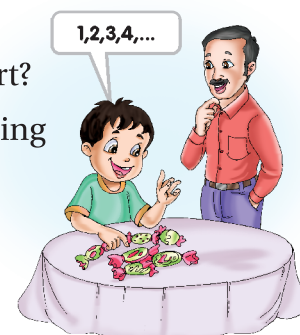


### Natural numbers

If you had to count the number of sweets your father gave you, how would you start? You would start counting with 1, then say 2, 3, 4,.....and so on. This list of counting numbers is infinite.

1, 2, 3, 4, 5,.....78,.....,100,.....,78891,..... are all counting numbers.

These counting numbers are called **natural numbers**.

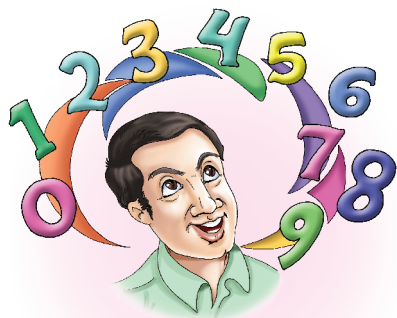


## Whole numbers

The natural numbers along with zero form the set of whole numbers. So, 0, 1, 2, 3, 4, 5,.....,34,.....,679,.....,1000,.....,35558907,.... are all **whole numbers**. Just like natural numbers, the list of whole numbers is infinite.

Are all natural numbers whole numbers? The answer would be 'yes'.

Will all whole numbers be natural numbers too? No, 0 is not a natural number.



So, now you know the basics of numbers. As a data analyst, you will have numbers juggling around in your head! Come on, join me! And as we juggle along, let me tell you a few properties of whole numbers.

### 1. Closure property

Let us take any two whole numbers and carry out the basic mathematical operations using them.

Add  
 $5 + 10 = 15$

Subtract  
 $5 - 10 = \text{not a whole number}$

Multiply  
 $5 \times 10 = 50$

Divide  
 $5 \div 10 = \frac{1}{2}$

When you add or multiply two whole numbers, the answer is always a whole number. This means that whole numbers are closed under multiplication and addition.

This is known as the **closure property** of whole numbers. When you subtract or divide two whole numbers, the answer is not always a whole number. This means that whole numbers are not closed under subtraction and division.

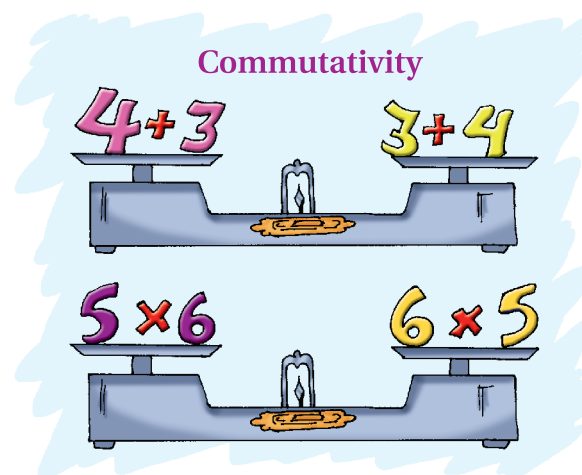
### 2. Commutative property

Let us do some simple calculations.

Addition:  $4 + 3 = 7$  &  $3 + 4 = 7$

Multiplication:  $5 \times 6 = 30$  &  $6 \times 5 = 30$

When you add or multiply two whole numbers in any order, the answer remains the same.



Since  $4 + 3 = 3 + 4$  &  $5 \times 6 = 6 \times 5$

We say that **multiplication and addition are commutative for whole numbers.**

Subtraction:  $58 - 30 = 28$  but  $30 - 58 \neq 28$  ( $\neq$  stands for 'not equal to')

Division:  $15 \div 3 = 5$  but  $3 \div 15 \neq 5$

When you change the order of numbers in subtraction and division, you get different answers.

Since,  $58 - 30 \neq 30 - 58$  &  $15 \div 3 \neq 3 \div 15$

**so, subtraction and division are not commutative for whole numbers.**

### 3. Associative property

Let us see what happens when we do these calculations:

$$(4 + 5) + 2 = 11 \quad \& \quad 4 + (5 + 2) = 11$$

$$(4 \times 5) \times 2 = 40 \quad \& \quad 4 \times (5 \times 2) = 40$$

This means that while adding and multiplying, we can group or associate numbers in any order.

i.e.  $(4 + 5) + 2 = 4 + (5 + 2)$

$$(4 \times 5) \times 2 = 4 \times (5 \times 2)$$

This is called **associativity of addition and multiplication.**

Let us try this rule for subtraction and division.

#### Subtraction

$$(20 - 5) - 4 = 15 - 4 = 11$$

$$20 - (5 - 4) = 20 - 1 = 19$$

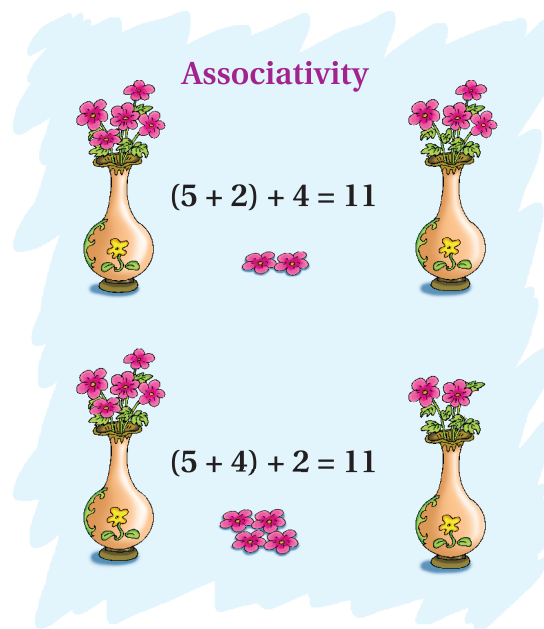
So,  $(20 - 5) - 4 \neq 20 - (5 - 4)$

#### Division

$$(20 \div 4) \div 5 = 5 \div 5 = 1$$

$$20 \div (4 \div 5) = 20 \div \frac{4}{5} = 25$$

So,  $(20 \div 4) \div 5 \neq 20 \div (4 \div 5)$



**Therefore, associativity does not hold for subtraction and division of whole numbers.**

#### 4. Distributive property

Consider the following multiplications:

$$7 \times 8 = 56, \quad 7 \times 3 = 21 \quad \& \quad 7 \times 5 = 35$$

Did you find that  $7 \times 8 = (7 \times 3) + (7 \times 5)$  ?

We can write 8 as the sum of 3 and 5.

$$\text{So, } 7 \times (5 + 3) = (7 \times 5) + (7 \times 3)$$

We use the distributive property to make calculations easier.

This is known as the **distributivity of multiplication** over addition.

We could write 8 as  $(10 - 2)$  as well.

$$\text{Then, } 7 \times (10 - 2) = (7 \times 10) - (7 \times 2)$$

This is known as the **distributivity of multiplication** over subtraction.

Let me tell you about two special numbers. They are 0 and 1. Can you guess why?

- Add 0 to any number and you get the same number.
- Multiply 1 with any number and you get the same number.

0 is the additive identity

1 is the multiplicative identity

What you need to do...



Play this fun game in groups. This card game, given in your follow-up journal, is about matching mathematical expressions to the correct property of numbers. The game will test how well you know your number properties.



## What you have learnt...



- ➔ Natural numbers are the counting numbers.
- ➔ Natural numbers together with zero form whole numbers.
- ➔ Whole numbers are closed under addition and multiplication.
- ➔ The commutative property and associative property hold true for addition and multiplication of numbers.
- ➔ Distributive property of multiplication over addition and subtraction makes calculations easier.

## What you need to know...



### Estimation

**As a data analyst, you would often need to estimate your numbers before drawing conclusions. In addition, when you calculate mentally, you need to estimate large numbers, isn't it? So, it is really important to know estimation well.**



Say you have ₹ 15,342 with you. Your brother asks you for ₹ 5,675 and the same day your friend asks you for ₹ 7,231. Now, if you want to quickly calculate how much money you will be left with, what will you do? You will estimate the amounts, won't you?

Estimating to the nearest thousand:

5,675  $\longrightarrow$  6,000

7,231  $\longrightarrow$  7,000

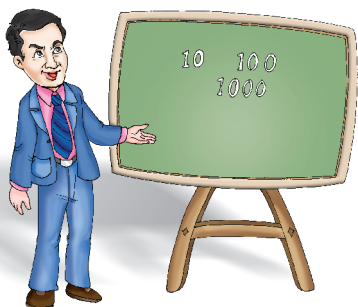
Estimated sum  $\longrightarrow$  13,000

The estimated amount you have with you is 15,000

Subtract the estimated sum from the estimated amount.

$$15,000 - 13,000 = 2,000$$

So, you will have approximately ₹ 2,000 left with you.



**We could have estimated the above numbers to the nearest hundreds or tens. Then, we would have got a better approximation of the correct amount. However, that would have taken more time to calculate.**

If the numbers are very large, it is not wise to estimate to the nearest hundreds or tens. Estimation should be done, based on the **size of the numbers**, the **accuracy required** and also the **time available**.

Say we have to estimate  $57 \times 122$

We round off 57 to 60 (round to the nearest 10)

Round off 122 to 120 (round to the nearest 10)

Multiply 60 and 120

Estimated answer is 7200

Now, use your calculator to find the actual answer.

### What you need to do...



The parking lot of a new conference hall has a capacity for 37,865 cars, and today, there are 21,376 such parking lots in the world. Approximately how many cars can these parking lots accommodate?

Also, find the exact answer by using a calculator.

### What you have learnt...



- ➔ When we need to make a quick calculation of something, we estimate the numbers.
- ➔ The place value to which we estimate depends on the size of the numbers, the accuracy required and the time available.
- ➔ When we solve questions with large numbers, it is a good idea to first estimate the numbers and then solve the question.



**Let me tell you something very exciting about numbers that will help you calculate sums in a jiffy. Have you heard of Vedic math? Okay, let us see what it is all about.**

Vedic math is our ancient system of mathematics, which can be used to solve various mathematical problems mentally. Let us solve each sum using Vedic math and then validate the answer with the help of a calculator. Which method do you think will be faster?

## Vedic Math

### To subtract 4,657 from 10,000

Subtract the digit in the ones place (7 in this case) from 10 and all the other digits from 9.

We get  $10,000 - 4,657 = 5,343$ .

### To multiply a number ending in 5 with itself

$$125 \times 125$$

Multiply the digits in the ones place.

$5 \times 5 = 25$ . These digits form the last two digits of the answer.

Multiply the number formed by the remaining digits (12) by its successor (13).

$12 \times 13 = 156$ . These are the first three digits from the left side in the answer.

Thus, we get 15,625

So,  $125 \times 125 = 15,625$ .

### What is $35 \times 35$ ?

Since,  $5 \times 5 = 25$

and  $3 \times 4 = 12$

So, my answer is 1,225.

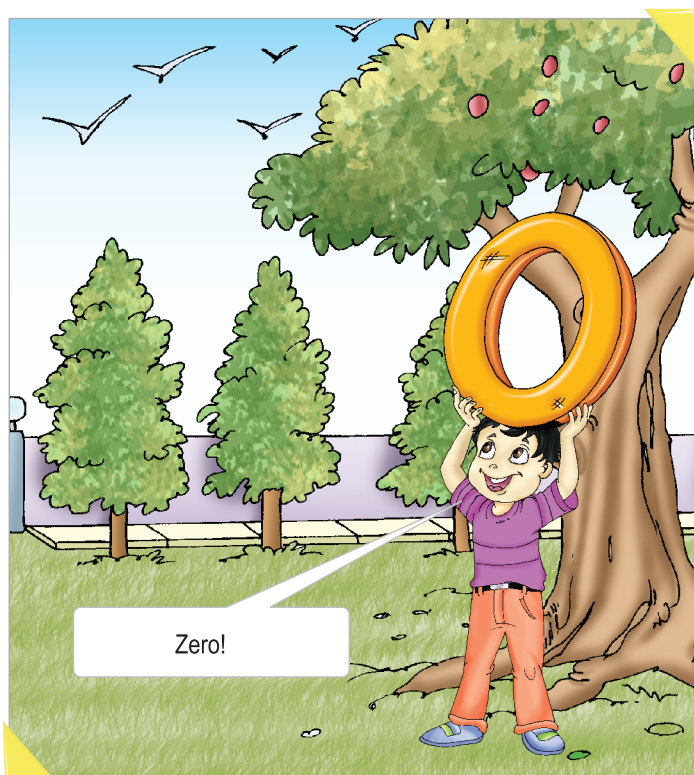
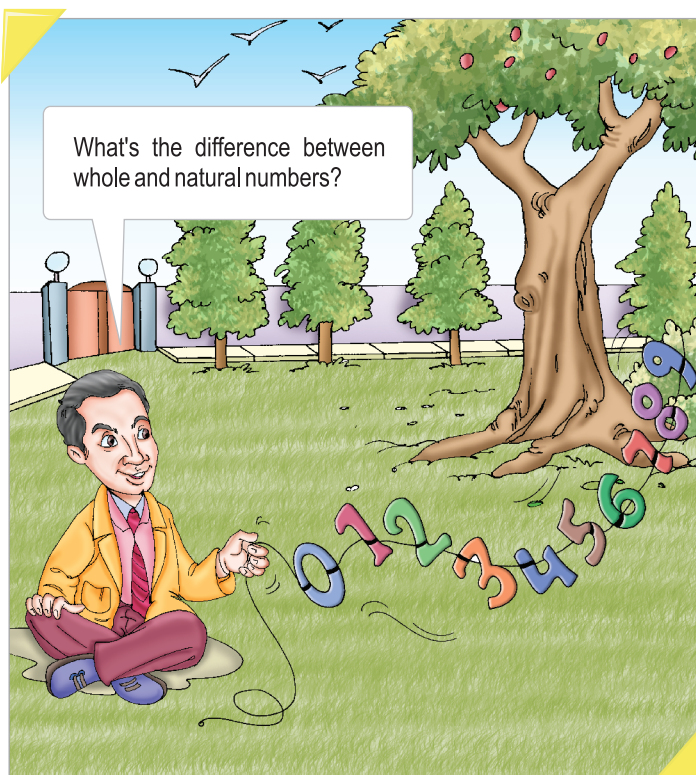
### What is $75 \times 75$ ?

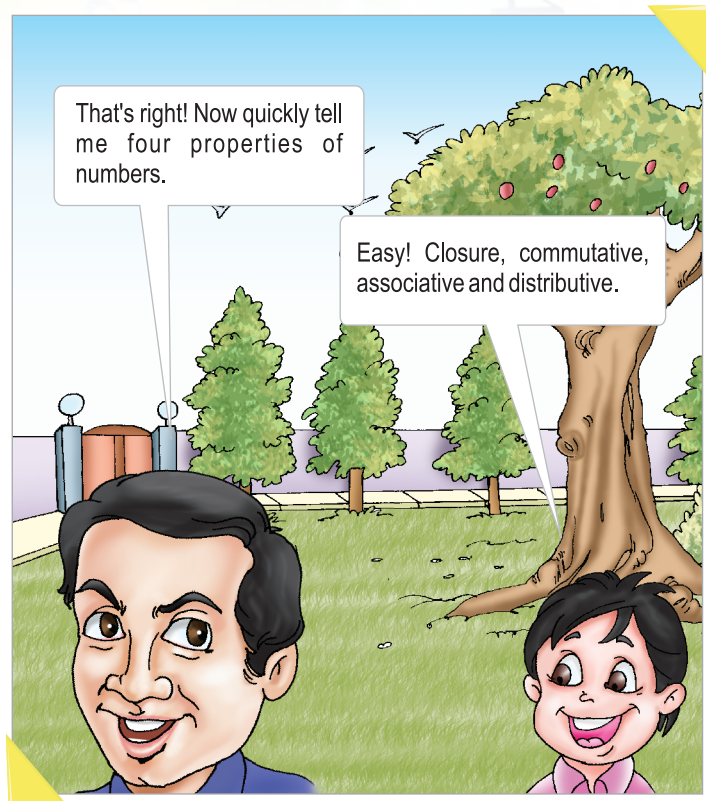
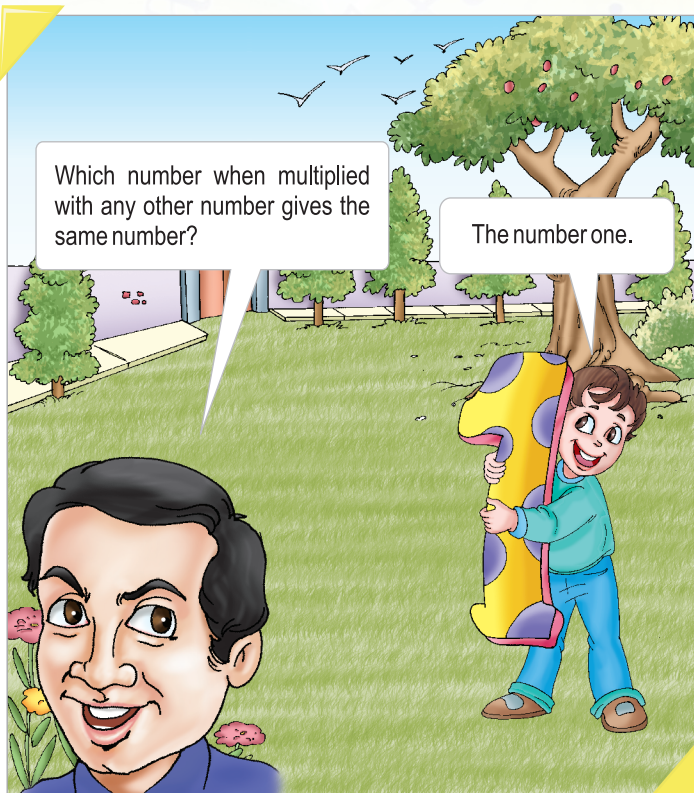
Since,  $5 \times 5 = 25$

and  $7 \times 8 = 56$

So, my answer is 5,625.

## What you need to remember...





### Time to work out...



1. Define natural and whole numbers.
2. Name the four basic properties of numbers.
3. Name the additive identity and the multiplicative identity of numbers.
4. State the factors that determine the place to which we estimate our numbers.
5. Give examples to show the following properties of whole numbers:
  - a. Closure with respect to addition and subtraction
  - b. Commutative with respect to addition and subtraction
  - c. Associative with respect to addition and subtraction
  - d. Distributivity of multiplication over addition and subtraction
6. Estimate 3,75,658 to the nearest:
  - a. Ten
  - b. Hundred
  - c. Thousand
  - d. Ten thousand
  - e. Lakh

7. Are the following equations correct?

- a.  $(45 - 4) - 7 = 45 - (4 - 7)$       b.  $46 \times 4 = 4 \times 46$   
c.  $6 + (9 + 3) = (6 + 9) + 3$       d.  $56 - 8 = 8 - 56$

8. Use the distributive property to solve these:

- a.  $35 \times 42$       b.  $74 \times 55$       c.  $(46 \times 2) + (46 \times 8)$       d.  $(58 \times 75) + (58 \times 25)$

9. If Sachin Tendulkar has so far scored 7,845 runs in test matches, how many more runs does he need to make 10,000 runs. Find the answer using Vedic math.
10. Find the difference between the largest and the smallest numbers formed by using the digits 6, 3, 0, 8, 2 each being used only once.
11. If an encyclopaedia has 4,789 pages and you read 87 such encyclopaedias, how many pages would you read?
12. Write two multiplication sums each in two and three digits. Now, estimate the answers and then check these answers using a calculator. Compare the two answers.
13. Make the chart for Indian place value system upto eight places.

### For the apprentice...

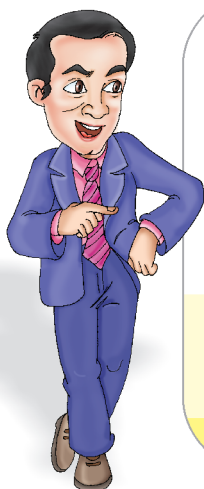


As an apprentice to a data analyst, you need to use the abacus to add numbers and make measurements on a rectangular tile, a triangular tile, a cuboid and a grid. Then, you would do some calculations and analyse the results. Finally, identify the properties of whole numbers.

### Time to part...!!!



**I am sure you are already wondering what it would be like to be a data analyst when you grow up! Do not forget to practice your numbers well. So look up these links on the Internet. You can also find out more about data analysts as well as about me on these sites. So, see you there...!!**



### Happy Surfing.....Click.....Click...

Find fascinating number facts at  
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